Discrete Mathematics I

B. Math. II

Back-Paper Examination

Instructions: All questions carry ten marks. All graphs are assumed to be simple.

1. Let $n > 2$ be an integer. Give an example of an n-regular graph which has no spanning subgraph that is $n-1$-regular.
2. Prove or disprove: If a graph G has no cut edge, then any two distinct vertices v, w are contained in a cycle.
3. If (X_1, Y_1) and (X_2, Y_2) are minimum cuts in a transportation network, then prove that $(X_1 \cup X_2, Y_1 \cap Y_2)$ is also a minimum cut.
4. Prove that a graph is bi-partite if and only if it does not contain a cycle with odd number of vertices.
5. Let G be a graph on 10 vertices that is NOT connected. Prove that G has at most 36 edges. Can equality hold?
6. Let $k \leq n$ be two integers. Prove that a $k \times n$ Latin rectangle can be completed to a Latin squaye of order n.
7. Let $N(n)$ denote the maximum number of pairwise orthogonal Latin squares of order n. Prove that if $n > 1$, then $N(n) \leq n - 1$.
8. Prove that in any non-trivial Steiner system $S(t, k, v)$, we must have
$$v \geq (t+1)(k-t+1)$$
.
9. Let $t \geq 2$. For any t design with b blocks and v points, prove that $b \geq v$.
10. Let n be a prime power. Construct a $2-(n^2+n+1, n+1, 1)$ design.